Nonlinear dual-core photonic crystal fiber couplers

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We study nonlinear modes of dual-core photonic crystal fiber couplers made of a material with the focusing Kerr nonlinearity. We find numerically the profiles of symmetric, antisymmetric, and asymmetric nonlinear modes and analyze all-optical switching generated by the instability of the symmetric mode. We also describe elliptic spatial solitons controlled by the waveguide boundaries.

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Photonic crystal fibers (PCFs) have attracted a lot of attention due to their intriguing properties, potential applications, and the development of successful fabrication technologies.[1] PCFs are characterized by a conventional cylindrical geometry with a two-dimensional lattice of airholes running parallel to the fiber axis. Such structures share many properties of photonic crystals, associated with the existence of frequency gaps where the light transmission is suppressed due to Bragg scattering, as well as the guiding properties of conventional optical fibers, due to the presence of a core in the structure.

Recent theoretical and experimental results reported the study and fabrication of dual-core PCF structures for broadband directional coupling or polarization splitting.[2–6] In PCFs, light confinement is restricted to the core of the fiber and therefore nonlinear effects, such as light self-trapping and localization in the form of spatial optical solitons,[7] become important. In particular, similar to two-dimensional nonlinear photonic crystals,[8] a PCF can support and stabilize both fundamental and vortex spatial optical solitons.[9,10] In sharp contrast with an entirely homogeneous nonlinear Kerr medium where spatial solitons are unstable and may collapse, it was shown that the periodic structure of PCF can stabilize the otherwise unstable two-dimensional solitons.

In this Letter, we make a further step forward in the study of nonlinear effects in the PCF geometry and analyze the existence and stability of nonlinear guided modes and spatial solitons in dual-core PCF couplers. The beam propagation and power-dependent switching in nonlinear directional couplers have been analyzed for the planar waveguide geometry.[11] Here we generalize those results for PCFs, as well as study the existence and stability of guided modes and elliptic spatial solitons controlled by the PCF holes. In particular, we find numerically the profiles of symmetric, antisymmetric, and asymmetric nonlinear modes and analyze all-optical switching based on the mode instability.

We consider a simple model of PCF that describes, at a given frequency, the spatial distribution of light in a nonlinear dielectric material with a triangular lattice of airholes with radius r in a circular geometry. We assume that the PCF material has a nonlinear Kerr response, and there are two missing holes at the center filled by the same material creating a nonlinear defect, as shown in Figs. 1(a)–1(c). In the substrate material, the linear refractive index is n_s, whereas inside the holes it is n_a. In the nonlinear regime, the light distribution in PCF is described by the equation

$$-i\frac{\partial E}{\partial z} = \Delta_\perp E + W(x,y)E + V(x,y)|E|^2E, \qquad (1)$$

where $W(x,y)=n_a+(n_s-n_a)V(x,y)$, $\Delta_\perp=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the transverse Laplacian, E is the normalized electric field, and $V(x,y)$ is an effective potential describing two solid cores in the lattice of holes. We normalize $V=1$ in the material and $V=0$ in the holes.

To find stationary nonlinear modes of PCF, we look for solutions in the form $E(x,y,z)=u(x,y)\exp(i\beta z)$ and obtain the nonlinear eigenvalue problem

$$\beta u = \Delta_\perp u + W(x,y)u + V(x,y)|u|^2u. \qquad (2)$$

To find the solutions of Eq. (2) for nonlinear localized modes, we consider a rectangular domain of the (x,y) plane and apply a finite-difference scheme, taking, respectively, N and M uniformly distributed samples x_i $(0\leq i<N)$ and y_j $(0\leq j<M)$ of the variables, as well as the corresponding samples for the stationary state, $u_{ij}=u(x_i,x_j)$ and the potential $V_{ij}=V(x_i,y_j)$. Substituting these variables into model (2) and imposing homogeneous boundary conditions in all four edges of the domain, we obtain an algebraic nonlinear problem of $2\times N\times M$ equations with the same number of unknown u_{ij}, which is finally solved by means of a globally convergent Newton method. The presence of the external linear potential given by two missing holes and the lattice of airholes makes the system nonscalable and its radial symmetry broken. Another

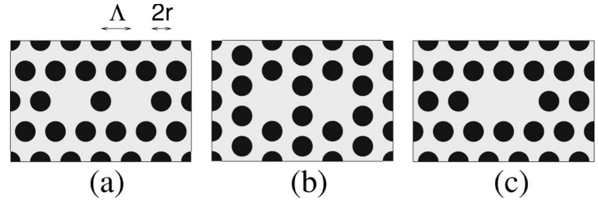

Fig. 1. Schematic of three studied designs of a nonlinear dual-core PCF. The coupler is created by two missing neighboring holes in a nonlinear material.

approach, which takes advantage of the lattice periodicity, was developed recently by Ferrando *et al.*[9]

First, we consider a single missing hole and find solutions numerically for the PCF spatial solitons.[9] Importantly, these stationary solutions are not perfectly radial, but they are stabilized by the PCF holes, in sharp contrast with the unstable self-trapped beams in nonlinear focusing Kerr media. To demonstrate this feature, we follow the standard analysis of the soliton stability[7] and analyze the soliton power as a function of the soliton propagation constant. A positive slope of this dependence indicates the soliton stability.

Next, we study the dual-core nonlinear PCFs shown in Figs. 1(a) and 1(b) and find the families of the spatially localized modes—the so-called PCF spatial twin solitons—as a function of the mode propagation number β. The corresponding solutions are similar for two cases of Figs. 1(a) and 1(b), and they can be envisaged as the modes of the effective dual-core fiber generated by the combined effect of the dual-core PCF refractive index and the nonlinear index induced by the mode amplitude itself. Contrary to the linear coupler, which supports only one symmetric mode and another antisymmetric one, the nonlinear dual-core PCF supports both symmetric (A) and antisymmetric (B) modes, along with an asymmetric mode (C), as shown in Fig. 2 for the case of Fig. 1(b). The corresponding spatial profiles of these nonlinear modes are shown in Fig. 2(d) as cross-sectional cuts along the line $y=0$.

To demonstrate the power relation between the modes, we follow the standard analysis of the soliton stability[12] and plot in Fig. 3 the soliton power as a function of the soliton propagation constant for the three different families. We notice that only two modes, symmetric and antisymmetric ones, may exist for low powers, whereas the asymmetric mode bifurcates from the symmetric mode at a certain threshold value of the mode power, above which the symmetric mode becomes unstable. Besides, the limit of these nonlinear modes when power vanishes corresponds to the linear coupler, so the power curves start at the values of β corresponding to the linear modes.

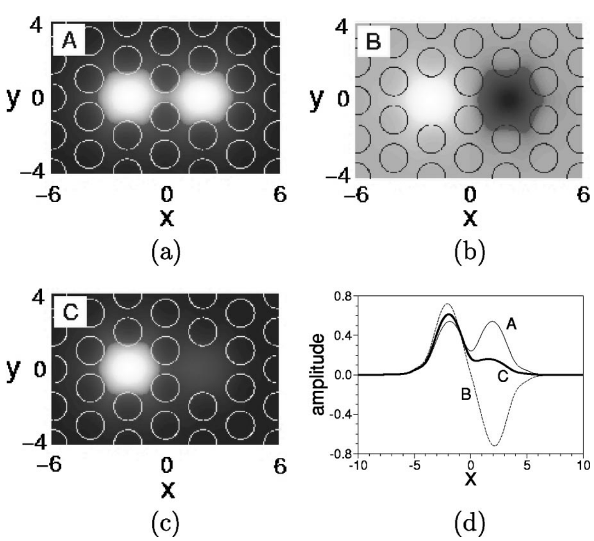

Fig. 2. (a)–(c) Light distribution between the two PCF cores in Fig. 1(b) for three distinct nonlinear modes: A, symmetric; B, antisymmetric; and C, asymmetric. (d) Transverse profiles of the nonlinear modes ($\beta=3.95$).

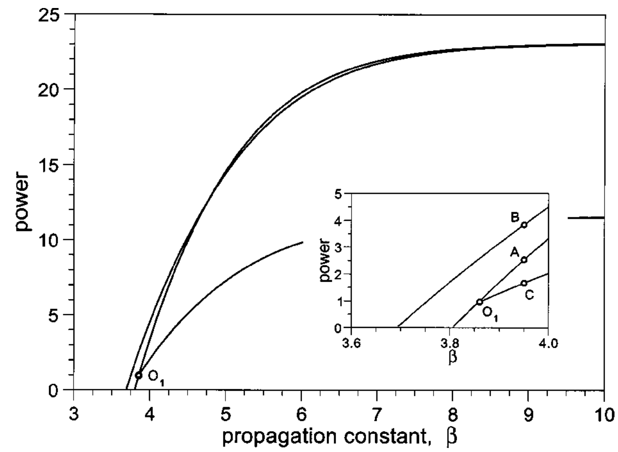

Fig. 3. Bifurcation diagram of the coupler modes. Inset, enlarged part near the bifurcation point. Asymmetric mode C bifurcates from symmetric mode A at point O_1 above a certain threshold in the mode power.

All the calculations were done for $n_s=5$, $n_a=0$, hole radius $r=0.75$, and hole separation $\Lambda=2$. The values of these parameters do not affect the physics of the system and were chosen for the sake of simplicity and to get a clear power diagram. In fact, a constant can always be added to both indices, only supposing a displacement in the propagation constant (the power curve would be shifted horizontally). On the other hand, the index difference between the substrate and the holes can always be compensated by a proper spatial rescaling of the system.

To study the switching properties of the nonlinear coupler, we carry out a series of numerical simulations using the standard beam propagation method. First, we calculate the stationary mode of a single-core PCF with the same parameters as the coupler corresponding to different powers. Then, this mode is launched into one of the cores of the dual-core coupler and, after some propagation, the output power is calculated at the same core. The propagation distance is determined considering the same coupler operating in the linear regime and propagating the linear single-core mode launched to one of the coupler cores up to the point where all the energy is transferred to the second core. For the parameters used in our examples this point is reached at $z=21.23$. The switching curve is obtained by varying the input power for the fixed propagation length, and it is plotted in Fig. 4. Similar to other types of nonlinear directional couplers, the input power is transferred completely to the second core for low powers, whereas it remains in the initial one for higher input powers. A change between both the regimes takes place in a relatively short range of powers, which constitutes a threshold where the power switching is triggered. For low powers (under the bifurcation point) the energy is completely transferred to the second core, although from the plot in Fig. 4 a residual amount seems to remain in the first one. This nonzero behavior of the curve

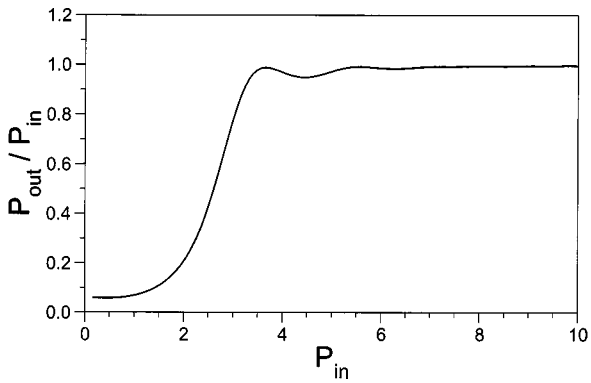

Fig. 4. Switching curve calculated for the PCF nonlinear coupler of Fig. 1(b). Due to the stability of the asymmetric mode and the instability of the symmetric one, the light launched into one core only does not switch to the second core but remains in the same core.

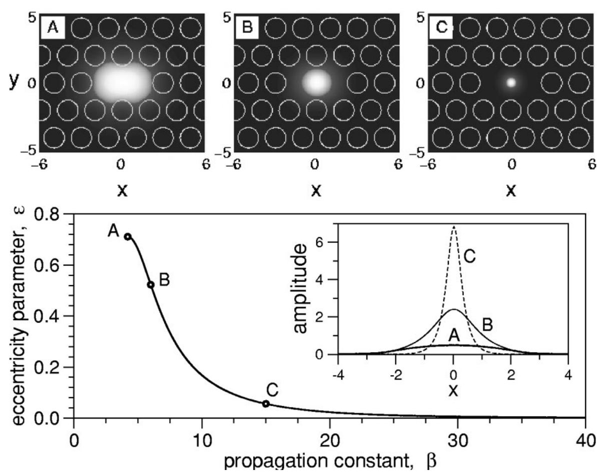

Fig. 5. Transformation of the elliptic guided modes to circular solitons for the growing mode power. Three examples on the top mark three points of the eccentricity curve for $\beta=4.2$, $\beta=6$, and $\beta=15$, respectively. Inset, the corresponding beam cross sections at $y=0$.

close to the origin is explained by the way in which the output power is computed, integrating the intensity in the semiplane corresponding to the desired core. In that way, the tail of the field in the other core slightly overlaps the first one due to its proximity, producing a residual power even when the field in the computed core is zero.

Finally, we study the third case of the closely spaced holes, shown in Fig. 1(c). In this case, there exists no bifurcation to the asymmetric state and the fundamental mode itself is elliptic as shown in the inset of Fig. 5. In the nonlinear case, this elliptic guided mode gives birth to an elliptic spatial soliton controlled by the boundaries of the holes. In Fig. 5, we plot the mode eccentricity parameter $\varepsilon=[1-(w_x/w_y)^2]^{1/2}$, where w_x and w_y are the mode axes (widths), and thus quantify the transformation of the elliptic guided modes (A) to the elliptic solitons (B) and then to the radially symmetric solitons (C). These results resemble the transformation of the shape of nonlinear guided modes in planar waveguides.[13]

In conclusion, we have demonstrated that several types of two-dimensional spatial optical solitons can be supported by a nonlinear dual-core PCF structure with the Kerr nonlinearity. We have analyzed numerically the existence and stability of symmetric, antisymmetric, and asymmetric nonlinear modes, demonstrating that the periodic refractive index of PCF provides an effective stabilization mechanism for these composite localized modes to exist in a nonlinear Kerr medium, in sharp contrast with an entirely homogeneous nonlinear Kerr medium where spatial solitons are known to be unstable and undergo collapse instability. We have studied all-optical switching in the nonlinear dual-core PCF coupler associated with the instability of the symmetric mode.

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